Mark Scheme 4734
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1	(i) $E(T) = E(X) + \lambda E(Y)$ $\Rightarrow 100 = 45 + 33\lambda$ $\Rightarrow \lambda = {}^5/_3$ AG	M1 A1	 **2**	Use $E(X + \lambda Y)$ aef
	(ii) $\text{Var}(T) = \text{Var}(X) + ({}^5/_3)^2\text{Var}(Y)$ $= 256$ $T \sim N(100, 256)$	M1 A1 B1√	 **3**	 ft variance
	(iii) Same student for X and Y so independence unlikely.	B1	**1**	Sensible reason
2	(i) Use $3a/2 = 1$	B1	**1**	Or similar
	(ii) $y = \frac{2}{3}x$ $y = 1 - \frac{1}{3}x$	B1 M1A1	 **3**	 M1 for correct gradient B1M1A0 if not y=…
	(iii) $\text{f}(x) = \begin{cases} \frac{2}{3}x & 0 \le x \le 1 \\ 1 - \frac{1}{3}x & 1 < x \le 3. \end{cases}$	B1√	**1**	ft (ii)
	(iv) $\int_0^1 \frac{2}{3}x^2\,dx + \int_1^3 (x - \frac{1}{3}x^2)\,dx$ $\left[\frac{2}{9}x^3\right]_0^1 + \left[\frac{1}{2}x^2 - \frac{1}{9}x^3\right]_1^3$ $= 4/3$	M1 A1√A1√ A1	 **4**	One correct, with limits ft from similar f aef
3	(i) Assumes breaking strengths have normal normal distributions Equal variances	B1 B1	 **2**	
	(ii) $H_0: \mu_T = \mu_U$, $H_1: \mu_T > \mu_U$ where μ_T, μ_U are means for treated and untreated thread.	B1		For both hypotheses
	$\bar{x}_T = 18.05, \quad \bar{x}_U = 17.26$	B1		May be implied below by 0.79
	$s_T^2 = 0.715, \quad s_U^2 = 0.738$	B1		Allow biased, 0.596, 0.590 if s^2
	$s^2 = (5\times0.715 + 4\times0.738)/9$	M1		$=(6\times0.596 + 5\times0.590)/9$
	EITHER: $(18.05 - 17.26)/[s\sqrt{(1/5+1/6)}]$ M1 $= 1.532$	 A1		With pooled variance est.
	Compare correctly with 1.383	M1		
	Reject H_0 and accept there is sufficient evidence that mean has increased so that the treatment has been successful.	A1√		Conclusion in context. Ft 1.532
	OR: $\bar{X}_T - \bar{X}_U \ge ks\sqrt{1/5 + 1/6}; = 0.713$	M1A1		Allow > or =
	$0.79 > 0.713$, reject H_0 etc	M1A1√	**8**	Or equivalent. Ft 0.713

4	(i) $s^2 = {}^1/_{11}(2604.4 - 177.6^2/12)$	M1		aef
	$= 1.0836\ldots$	A1		
	Use $\bar{x} \pm t\sqrt{\frac{s^2}{12}}$	M1		
	$t = 2.201$	B1		
	$\bar{x} = 177.6/12 = 14.8$	A1		
	(14.14,15.46) , (14.1, 15.5)	A1	**6**	
	(ii) EITHER:$(14.8 - 15.4)/(\sqrt{(s^2/12)})$	M1		With their variance
	$= -1.997$	A1		
	Compare correctly with –1.796	M1		
	Reject H_0 and accept that there is evidence that the mean is less than 15.4	A1√		In context. Ft – 1.997
	OR: $\bar{X} - 15.4 \le -k\sqrt{\frac{s^2}{12}}$; $\bar{X} \le 14.86$	M1A1		Allow < or =
	14.8 < 14.86, reject H_0 etc	M1A1√	**4**	Or equivalent. Ft 14.86
5	(i) 978/1200 = 0.815	B1	**1**	
	(ii) Use $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{1200}}$	M1		Reasonable variance
	$z = 1.645$	B1		
	$\sqrt{(0.815 \times 0.185/1200)}$	A1√		ft $\hat{p}$ Allow 1199
	(0.797,0.833)	A1	**4**	Interval
	(iii) If a large number of such samples were taken, p would be contained in about 90% of the confidence intervals.	B2	**2**	B1 if idea correct but badly expressed.
	(iv) $1.645\sqrt{(0.815 \times 0.185/n)} = 0.01$	M1		Allow one error; > or <
		A1		All correct
	$n = 1.645^2(0.815 \times 0.185)/0.01^2$	M1		Correct procedure for sim equ
	$= 4080$	A1	**4**	Integer rounding to 4100

6	(i) $\int_1^t \frac{3}{x^4}\,dx$	M1		Any variable
	$F(t) = \begin{cases} 1-\frac{1}{t^3} & t \ge 1, \\ (0 & \text{otherwise.}) \end{cases}$	A1	**2**	

(ii) $G(y) = P(Y \le y)$	M1		
$= P(T \le y^{1/3})$	A1		
$= F(y^{1/3})$	M1		
$= 1 - 1/y$	A1 √		ft $F(t)$
$g(y) = G'(y)$	M1		
$= 1/y^2,\ y \ge 1$ AG	A1	**6**	

(iii) EITHER $\int_1^\infty \frac{\sqrt{y}}{y^2}\,dy$ OR $\int_1^\infty \frac{3t^{\frac{3}{2}}}{t^4}\,dt$	M1		
$\left[-2y^{-1/2}\right]_1^\infty$ $\left[-2t^{-3/2}\right]_1^\infty$	B1		
$= 2$	A1	**3**	

7	(i)(a) H_0: Eye colour and reaction are not associated.	B1		Or equivalent (independent, or unrelated)
	H_1: Eye colour and reaction are associated	B1	**2**	
	(b) 65×39/140	B1	**1**	
	(c) $6.11^2/18.11 + 5.3^2/11.7 + 0.81^2/9.19$	M1		Or equivalent ; one correct
	2.061 + 2.401+ 0.071	A1		At least 3 dp here
	4.533, 4.53 AG	A1	**3**	But accept from 2 dp
	(d) $\nu = 4$	B1		Stated or implied
	Use tables to obtain α = 2½	B1	**2**	

(ii) H_0: $p_{BL} = p_{BR} = 0.4$, $p_O = 0.2$ (H_1: At least two prob. not as above)	B1		Or in words, in terms of probs or proportions
E values 56 56 28	M1A1		
$\chi^2 = 9^2/56 + 14^2/56 + 8^2/28$	M1		
= 5.839	A1		Accept 5.84
Compare correctly with 5.991	M1		M1A0 if 5.991 seen and consistent conclusion but the no explicit comparison
Accept that sample is consistent with the hypothesis.	A1√	**7**	
SR: If three tests for p then count only $p_{BR} = 0.4$.			
$(42/140 - 0.4)/\sigma$	M1		
$\sigma = \sqrt{(0.4\times0.6/140)}$; -2.415	A1A1		
Compare with –1..96; conclusion in context	M1A1		Max 6/7 (with H_0)